



## Multibody System Dynamics: MBDyn Overview



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# Outline

- **Multibody dynamics**
- **Software architectures**
- **Problems**
- **Arbitrary motion description**
- **Deformable components**
- **Solving the problem**
- **Extracting useful information**
- **Examples of multibody modeling with MBDyn**
- **Future development**
- **Documentation and support**

**Basic equations:**

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, t)$$

- **mechanics of unconstrained system of bodies**
- **subjected to configuration-dependent loads**

**Can be obtained from many (equivalent!) approaches:**

- ***Newton-Euler*: linear/angular equilibrium of each body**
- ***d'Alembert-Lagrange*: virtual work of active forces/moments**
- ***Gauss, Hertz, Hamilton, ...*: variational principles**

## Constrained system: kinematic constraints

- holonomic

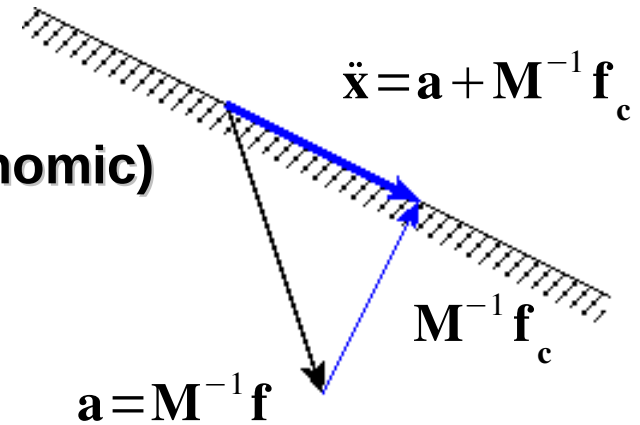
$$\phi(\mathbf{x}, t) = 0$$

- non-holonomic (not integrable to holonomic)

$$\psi(\mathbf{x}, \dot{\mathbf{x}}, t) = 0$$

usually

$$\mathbf{A}(\mathbf{x}, t) \dot{\mathbf{x}} = \mathbf{b}(\mathbf{x}, t)$$



- algebraic relationship between kinematic variables
- explicitly dependent on time: rheonomic
- scleronomous otherwise



**Minimal set:**

$$\mathbf{x} = \mathbf{x}(\mathbf{q}, t)$$

**usually, this relationship:**

- is not known in advance, or
- cannot be easily made explicit with respect to  $\mathbf{q}$

**Coordinate partitioning is required, e.g.:**

- direct elimination from derivative of constraint equation
- QR or similar decomposition

**Results in Maggi-Kane equations and similar approaches**

**Small system is obtained by expensive numerical reduction**  
**(unless topology knowledge can be exploited)**

**Redundant set:**

$$\delta(\lambda \cdot \phi) = \delta \lambda \cdot \phi + \delta \mathbf{x} \cdot \phi_{/x}^T \lambda$$

$$\delta(\mu \cdot \psi) = \delta \mu \cdot \psi + \delta \mathbf{x} \cdot \psi_{/\dot{x}}^T \mu$$

**By Lagrange multipliers:**

- dynamics of constrained system using physical coordinates
- constraint reactions applied to equations of motion
- algebraic constraints explicitly added to the system
- Multiple bodies with few actual dofs:
  - system size nearly doubles
- Multiple bodies with few constraints:
  - system size not significantly altered
- Sparsity is almost preserved



$$\begin{bmatrix} \mathbf{M} & \phi_{/x}^T \\ \phi_{/x} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{b}' \end{Bmatrix}$$

Constraint equations written “as is”:

$$\phi(\mathbf{x}, t) = 0$$

problem becomes differential algebraic (DAE); issues:

- needs specific care to be solved: (nearly) L-stable integration, i.e.
  - unconditionally stable, and
  - $\Delta \mathbf{x}_{k+1} \rightarrow 0$  for  $\Delta t \rightarrow \infty$
- the constraint equation implies the additional constraints

$$\dot{\phi}(\mathbf{x}, t) = 0$$

$$\ddot{\phi}(\mathbf{x}, t) = 0$$

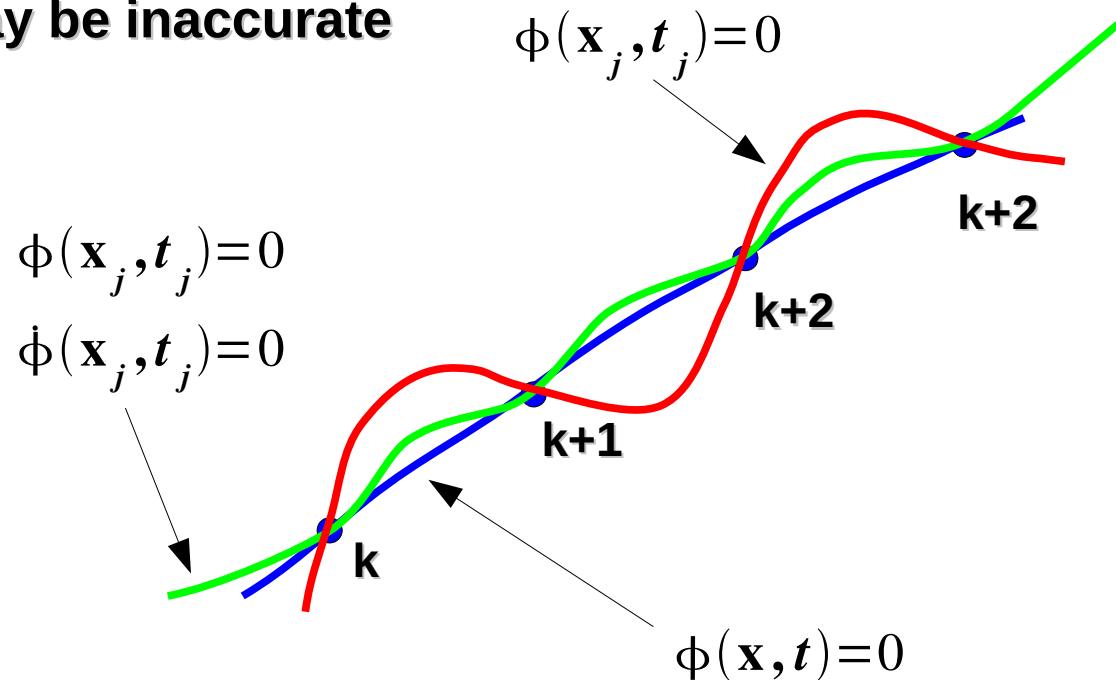
but they are not explicitly enforced:

may need constraint stabilization (Gear et al.)

# Multibody dynamics

## Constraint equations:

- $\mathbf{x}$  is correct
- derivatives may be inaccurate
- multipliers may be inaccurate





**Alternative: constraint equations differentiated to second order:**

$$\phi_{/x} \ddot{\mathbf{x}} = \mathbf{b}'$$

problem remains ordinary differential (ODE);

- can be solved by conditionally stable algorithms
- the constraint equation does not imply the original constraints

$$\phi(\mathbf{x}, t) = 0$$

$$\dot{\phi}(\mathbf{x}, t) = 0$$

**definitely needs constraint stabilization!**

common technique: Baumgarte

$$\phi_{/x} \ddot{\mathbf{x}} = \mathbf{b}' - 2\alpha \dot{\phi} - \beta^2 \phi$$

**(violation governed by asymptotically stable linear differential eq.)**



- **Monolithic:**
  - user prepares specific model using built-in library elements
  - general-purpose solver swallows model and spits results
- **Library:**
  - user writes specific solver using library elements
    - usually needs programming skills; the solver must be compiled
  - specific solver solves the problem and spits results
- **Symbolic manipulators:**
  - user writes equations
  - symbolic manipulation engine solves equations and spits results
- **Modelica (and Modelica-like):**
  - user prepares model using a modeling language and libs
  - general-purpose interpreter generates specific solver
  - specific solver solves the problem and spits results



Free software examples (surely there are more):

- **Monolithic:**

- **MBDyn**

non-free counterparts omitted

- **Library:**

- **DynaMechs (C++)**
- **Mbs3d (requires Matlab)**
- **Open Dynamics Engine (ODE) (C++)**

- **Symbolic manipulators:**

- **3D\_MEC**
- **EasyDyn (MuPad)**
- **RoboTran (requires Matlab)**

- **Modelica:**

- **OpenModelica?**

frequently architectures overlap



- **MBDyn is monolithic**
- **Input consists in a text file**
- **The input syntax allows some flexibility, e.g.:**
  - **math expressions evaluation**
  - **variables definition**
  - **“rigorous” syntax checking, but free style, indentation, ...**
- **Relevant portions of the code are modular and can be extended by:**
  - **writing run-time loadable modules**
  - **hacking the code (it's free, all in all!)**
- **There is no built-in pre-post processing facility**
- **Help in this area is warmly appreciated!**
  - **MBDyn output can be translated into EasyAnim**
  - **there is an independent, partial customization based on Blender**

**Equations of motion: for each node (purely geometrical entity),**

- **Newton-Euler, written as first-order system of equations:**

$$\begin{aligned}\mathbf{M} \dot{\mathbf{x}} &= \mathbf{p} \\ \dot{\mathbf{p}} &= \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, t)\end{aligned}$$

- **Momentum and momenta moment instead of pseudo-velocities**
- **allows multiple contributions to inertia of a single node**

**Constrained equations in differential-algebraic form:**

$$\begin{aligned}\mathbf{M} \dot{\mathbf{x}} &= \mathbf{p} \\ \dot{\mathbf{p}} + \phi_{/x}^T \lambda &= \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, t) \\ \phi(\mathbf{x}, t) &= 0\end{aligned}$$



- **Fundamental problem:**
  - **Integration of Initial Value Problem (IVP) in time**
- **Static analysis as degeneration of IVP dynamic analysis:**
  - **momentum and momenta moment definitions omitted**
  - **only gravity is considered**
  - **system determination only provided by kinematic constraints and deformable components**
- **Kinematic analysis as degeneration of IVP dynamic analysis:**
  - **inertia elements omitted**
  - **system determination only provided by kinematic constraints**
  - **deformable components can act as “regularization”**

# Problems

$\mathbf{K}' = \mathbf{I}$  : Moore-Penrose pseudo-inverse

## Experimental inverse dynamics problem

- inverse kinematics:

*same matrix!*

$$\begin{array}{l}
 \left[ \begin{array}{c} \mathbf{K}' \\ \phi_{/x} \end{array} \right] \begin{array}{c} \phi_{/x}^T \\ 0 \end{array} \left\{ \begin{array}{c} \Delta \mathbf{x} \\ \Delta \lambda_0 \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ -\phi(\mathbf{x}, t) \end{array} \right\} \\
 \left[ \begin{array}{c} \mathbf{K}' \\ \phi_{/x} \end{array} \right] \begin{array}{c} \phi_{/x}^T \\ 0 \end{array} \left\{ \begin{array}{c} \Delta \dot{\mathbf{x}} \\ \Delta \lambda_1 \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ \mathbf{b}(\mathbf{x}, t) \end{array} \right\} \\
 \left[ \begin{array}{c} \mathbf{K}' \\ \phi_{/x} \end{array} \right] \begin{array}{c} \phi_{/x}^T \\ 0 \end{array} \left\{ \begin{array}{c} \Delta \ddot{\mathbf{x}} \\ \Delta \lambda_2 \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ \mathbf{b}'(\mathbf{x}, \dot{\mathbf{x}}, t) \end{array} \right\}
 \end{array}$$

nonlinear  
linear  
linear

- the RHS contains the desired motion and its derivatives
- the (regularized) static analysis provides the kinematic inversion

$$\phi_{/x}^T \Delta \lambda = \mathbf{f}'(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}, t)$$

linear



- **Experimental direct eigenanalysis**
  - **issues with constraints formulation (mainly with rotations)**
  - **issues with equations implementation (matrices not available)**
  
- **Relative coordinate frame dynamics**
  - **imposed frame motion: modifications only to RHS inertia elems**
  - **instrumental for many helicopter rotor dynamics problems**





- **Mechanical degrees of freedom:**
  - **structural node positions in the absolute reference frame**
  - **structural node orientation with respect to the absolute frame**
- **Kinematics is always written with respect to the absolute frame**
- **Newton-Euler equations are written in the absolute frame**
  - **moment equilibrium (Euler) equations are written with respect to the respective (moving) node**
- **Special elements may introduce further approximations**
  - **e.g. Component Mode Synthesis (CMS) element**

# Arbitrary motion description

## Orientation handling:

- orientation variables: Cayley-Gibbs-Rodrigues parameters
- orientation matrix:

$$\mathbf{R} = \mathbf{R}(\mathbf{g})$$

- orthonormality:

$$\mathbf{R}^T = \mathbf{R}^{-1}$$

- derivative:

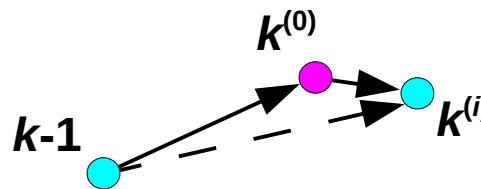
$$\dot{\mathbf{R}} \mathbf{R}^T = \boldsymbol{\omega} \times = (\mathbf{G}(\mathbf{g}) \dot{\mathbf{g}}) \times$$

- incremental approach from step  $k$  to  $k+1$  to eliminate the orientation parameters singularity issue (increments are necessarily small for accuracy):

$$\mathbf{R}_k = \mathbf{R}(\mathbf{g}_k) \mathbf{R}_{k-1}$$

# Arbitrary motion description

- **Orientation handling:**
  - the actual orientation variables are the Cayley-Gibbs-Rodrigues parameters relative to the correction phase of each step
  - $k$ : time step counter
  - $i$ : correction iteration counter (0: predicted value)



- **Orientation matrix:**

$$\mathbf{R}_k^{(i)} = \mathbf{R}(\mathbf{g}_\Delta^{(i)}) \mathbf{R}_k^{(0)}$$

- **Derivative:**

$$\dot{\mathbf{R}}_k^{(i)} (\mathbf{R}_k^{(i)})^T = \omega_k^{(i)} \times = (\mathbf{R}(\mathbf{g}_\Delta^{(i)}) \omega_k^{(0)}) \times + (\mathbf{G}(\mathbf{g}_\Delta^{(i)}) \dot{\mathbf{g}}_\Delta^{(i)}) \times$$



- **Incremental orientation from previous step:**
  - **Orientation parameters order of magnitude:**

$$\mathbf{g} \sim \mathbf{O}(\|\boldsymbol{\omega}\| \Delta t)$$

- **Incremental orientation from prediction (as in MBDyn):**
  - **Orientation parameters order of magnitude:**

$$\mathbf{g}_{\Delta} \sim \mathbf{O}(\Delta t^{n+1})$$

where  $n$  is the min between the order of the predictor and of the integration method  
(MBDyn: 3 and 2, respectively, so  $n = 2$ )

- **As a consequence:**

$$\mathbf{R}(\mathbf{g}_{\Delta}) \approx \mathbf{I}$$

$$\mathbf{G}(\mathbf{g}_{\Delta}) \approx \mathbf{I}$$

(only in Jacobian)

$$\dot{\mathbf{G}}(\mathbf{g}_{\Delta}) \approx 0$$

# Deformable components

- **Lumped deformable components**
  - rod (1D)
  - linear, angular components (3D)
  - linear & angular component (6D)
- **Intrinsic, composite-ready Finite-Volume beam element**
  - arbitrary constitutive law
  - piezoelectric constitutive law
  - aerodynamic beam element
- **Intrinsic, composite-ready shell and membrane elements**
- **Component Mode Synthesis (CMS)**
  - attached to a floating frame (a node)
  - linear state-space representation of unsteady aerodynamics

## Lumped deformable components (3D, 6D):

$$\theta = \mathbf{ax}(\exp^{-1}(\mathbf{R}_a^T \mathbf{R}_b))$$

$$\mathbf{m} = \mathbf{R}(\xi \theta) \tilde{\mathbf{m}}(\theta)$$

- **Attached form:**  $\xi=0, \xi=1$ 
  - **Constitutive properties referred to either of the connected nodes**
- **Intrinsic form (invariant:  $\xi=1/2$ ):**
  - **Constitutive properties referred to a floating reference frame**
  - **Intrinsically handles geometrical nonlinearity related to rotations**
  - **Correctly captures bending-torsion buckling behavior**
  - **Essential for anisotropic deformable components**

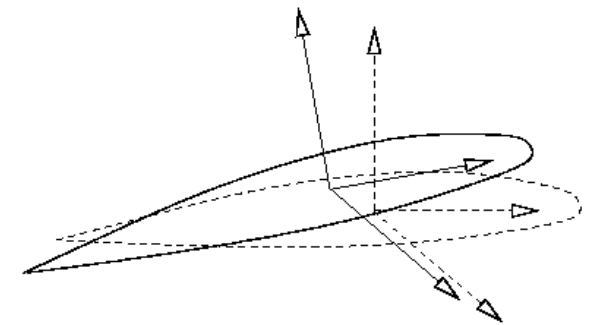
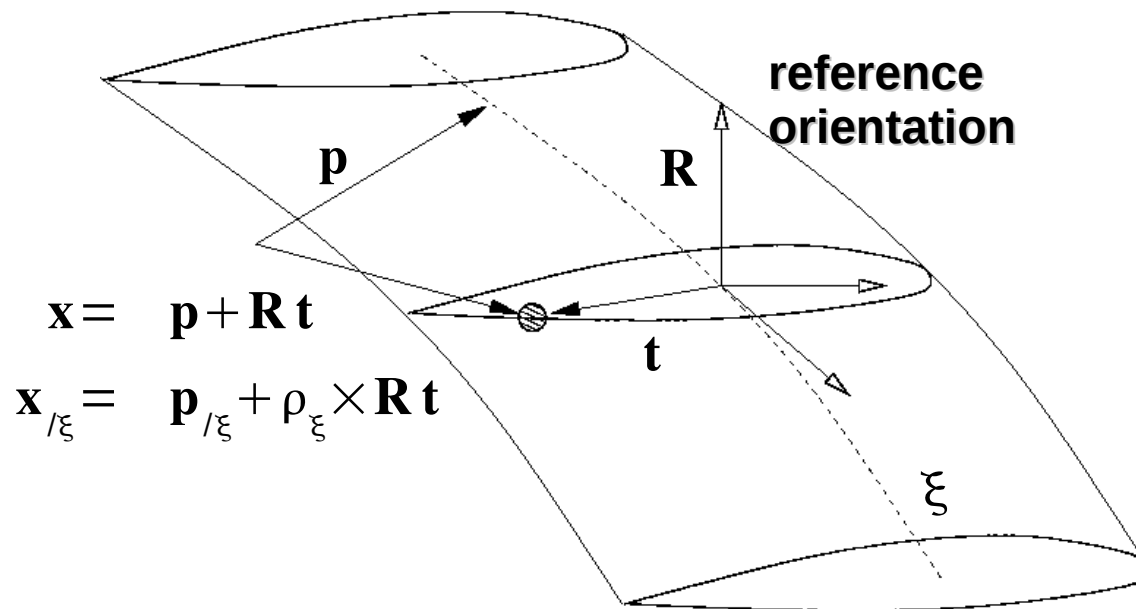
# Deformable components

## Intrinsic, composite-ready beam

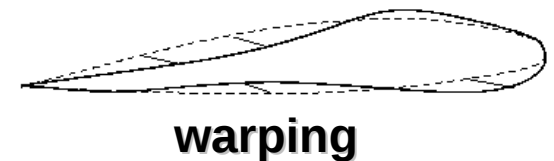
- Topology:
  - 1D reference line  $\mathbf{p}$ , 1D reference structure  $\mathbf{R}$
  - 2D section characterization

reference motion:  $\mathbf{p}$ ,  $\mathbf{R}$

reference line



+



warping

# Deformable components

## Intrinsic, composite-ready beams

- strain measure:

$$\nu = \mathbf{R}^T \mathbf{p}_{/\xi} - \mathbf{R}_0^T \mathbf{p}_{0/\xi}$$

$$\kappa = \mathbf{R}^T \rho_{\xi} - \mathbf{R}_0^T \rho_{\xi 0}$$

- equilibrium (from VWP):

$$\mathbf{f}_{/\xi} = \tau$$

$$\mathbf{m}_{/\xi} + \mathbf{p}_{/\xi} \times \mathbf{f} = \mu$$

- constitutive properties:

$$\mathbf{f} = \mathbf{f}(\nu, \kappa)$$

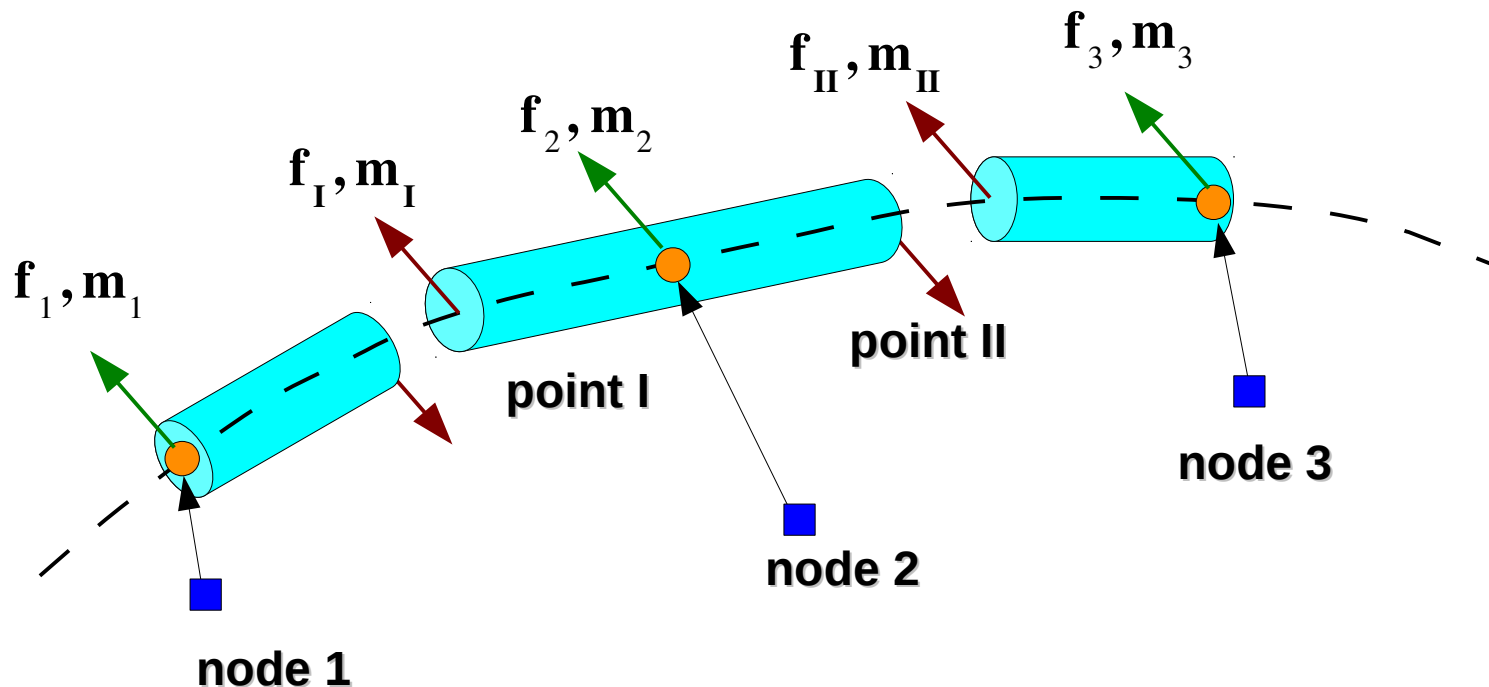
$$\mathbf{m} = \mathbf{m}(\nu, \kappa)$$



# Deformable components

## Intrinsic, composite-ready beams: 3-node discretization

- **Finite Volume approach: equilibrium of finite portions of beam**
- **internal forces function of node kinematics thru constitutive laws**
- **warping goes into constitutive properties computation**





## Numerical integration

- **implicit, (quasi-)L stable 2 step algorithm**

$$\mathbf{y}_k = \mathbf{a}_1 \mathbf{y}_{k-1} + \mathbf{a}_2 \mathbf{y}_{k-2} + \Delta t (\mathbf{b}_0 \dot{\mathbf{y}}_k + \mathbf{b}_1 \dot{\mathbf{y}}_{k-1} + \mathbf{b}_2 \dot{\mathbf{y}}_{k-2})$$

- **tunable algorithmic dissipation: asymptotic spectral radius  $1 \rightarrow 0$** 
  - **asymptotic spectral radius = 0: 2<sup>nd</sup> order BDF**
  - **“optimal” dissipation: spectral radius  $\sim 0.6$**
- **second-order accurate, with third-order accurate predictor**
- **variable time step**
- **not ideal for non-smooth problems (multi-step)**
- **different integrators can be used; new ones can be implemented**

# Solving the problem

- Prediction:**

$$\dot{\mathbf{y}}_k^{(0)} = (\mathbf{m}_1 \mathbf{y}_{k-1} + \mathbf{m}_2 \mathbf{y}_{k-2}) / \Delta t + \mathbf{n}_1 \dot{\mathbf{y}}_{k-1} + \mathbf{n}_2 \dot{\mathbf{y}}_{k-2}$$

$$\mathbf{y}_k^{(0)} = \mathbf{a}_1 \mathbf{y}_{k-1} + \mathbf{a}_2 \mathbf{y}_{k-2} + \Delta t (\mathbf{b}_0 \dot{\mathbf{y}}_k^{(0)} + \mathbf{b}_1 \dot{\mathbf{y}}_{k-1} + \mathbf{b}_2 \dot{\mathbf{y}}_{k-2})$$

- Correction iteration:**

$$\mathbf{f}_{/\dot{\mathbf{y}}} \Delta \dot{\mathbf{y}}^{(i)} + \mathbf{f}_{/y} \Delta \mathbf{y}^{(i)} = -\mathbf{f}(\dot{\mathbf{y}}_k^{(i-1)}, \mathbf{y}_k^{(i-1)}, \mathbf{t}_k)$$

but

$$\Delta \mathbf{y}^{(i)} = \Delta t \mathbf{b}_0 \Delta \dot{\mathbf{y}}^{(i)}$$

**the problem becomes algebraic**

$$(\mathbf{f}_{/\dot{\mathbf{y}}} + \Delta t \mathbf{b}_0 \mathbf{f}_{/y}) \Delta \dot{\mathbf{y}}^{(i)} = -\mathbf{f}(\dot{\mathbf{y}}_k^{(i-1)}, \mathbf{y}_k^{(i-1)}, \mathbf{t}_k)$$

$$\dot{\mathbf{y}}_k^{(i)} = \dot{\mathbf{y}}_k^{(i-1)} + \Delta \dot{\mathbf{y}}^{(i)}$$

$$\mathbf{y}_k^{(i)} = \mathbf{y}_k^{(i-1)} + \Delta t \mathbf{b}_0 \Delta \dot{\mathbf{y}}^{(i)}$$

# Solving the problem

## Model assembly

- model could be input incorrectly
- initial values of the state (position, velocity, reactions) are needed
- this might not be a trivial task
- initial state values must comply with constraints:

$$\phi(\mathbf{x}_0, t_0) = 0$$

$$\dot{\phi}(\mathbf{x}_0, t_0) = 0$$

- a dummy static nonlinear problem is solved (regularization):

$$\mathbf{K}'(\mathbf{x} - \mathbf{x}_0) + \phi_{/x}^T \lambda' = \mathbf{f}'$$

$$\mathbf{C}'(\dot{\mathbf{x}} - \dot{\mathbf{x}}_0) + \phi_{/x}^T \mu' = \dot{\mathbf{f}}'$$

$$\phi(\mathbf{x}, t_0) = 0$$

$$\dot{\phi}(\mathbf{x}, t_0) = 0$$

## Solving the problem

### Solution initialization (so-called “derivatives”)

- explicit problem:

$$\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, t)$$

- implicit problem:

$$0 = \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}, t)$$

- modified correction phase to initialize solution:

$$(\mathbf{f}_{/\dot{\mathbf{y}}} + c \mathbf{f}_{/\mathbf{y}}) \Delta \dot{\mathbf{y}}^{(i)} = -\mathbf{f}(\dot{\mathbf{y}}_0^{(i-1)}, \mathbf{y}_0, t_0)$$

$$\dot{\mathbf{y}}_0^{(i)} = \dot{\mathbf{y}}_0^{(i-1)} + \Delta \dot{\mathbf{y}}^{(i)}$$

$$\mathbf{y}_0 = \mathbf{y}_0$$

- convergence no longer quadratic, but saves lots of code duplication
- Setting  $c=0$  might not work (problem can be structurally singular)

## Extracting useful information

- Detailed analysis requires detailed models, but...
- excessive details endanger the chance to extract useful information
- Proper Orthogonal Decomposition allows to extract information from redundant measures
- Consider a set of  $N$  measurements  $\mathbf{X}$  for  $n$  time steps; their SVD:

$$\mathbf{X}^T (\in \mathbb{R}^{n \times N}) = \mathbf{U} \Sigma \mathbf{V}^T$$

- The singular values allow to determine the  $m$  most relevant signals

$$\mathbf{X}_{1:m,n}^T = \mathbf{U}_{n,1:m} \Sigma_{1:m,1:m} \mathbf{V}_{N,1:m}^T$$

- Note that

$$\begin{aligned} \mathbf{X}^T \mathbf{X} &= \mathbf{U} \Sigma^2 \mathbf{U}^T & \mathbf{X} \mathbf{X}^T &= \mathbf{V} \Sigma^2 \mathbf{V}^T \\ \mathbf{U}^T \mathbf{X}^T &= \Sigma \mathbf{V}^T & \mathbf{X}^T \mathbf{V} &= \mathbf{U} \Sigma \end{aligned}$$

- This allows to efficiently compute the singular values and the POMs



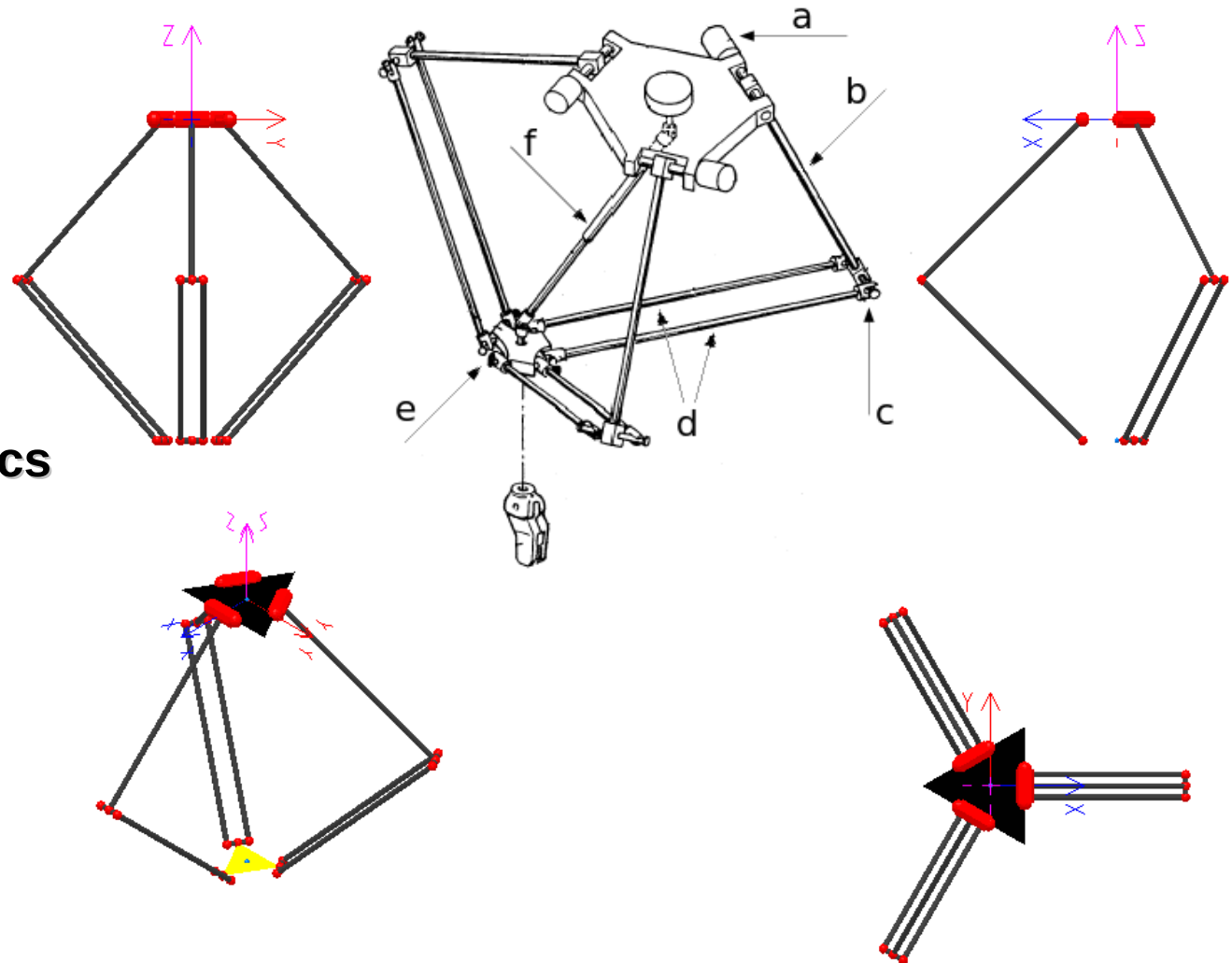
- **The POMs can be used to identify a transition matrix**

$$\mathbf{X}^{(k+1)} = \Phi \mathbf{X}^{(k)}$$

- **If  $\mathbf{X}$  contains the free response of the system, the transition matrix allows to estimate the relevant eigenvalues (AR model)**
- **More sophisticated system identification techniques can be used (model order reduction is an open research field)**
- **A technique based on covariance estimates from time histories has been recently proposed; works for:**
  - **free response**
  - **forced response**
  - **unmeasured forced response**

# Examples of multibody modeling with MBDyn

## Robotics: delta robot

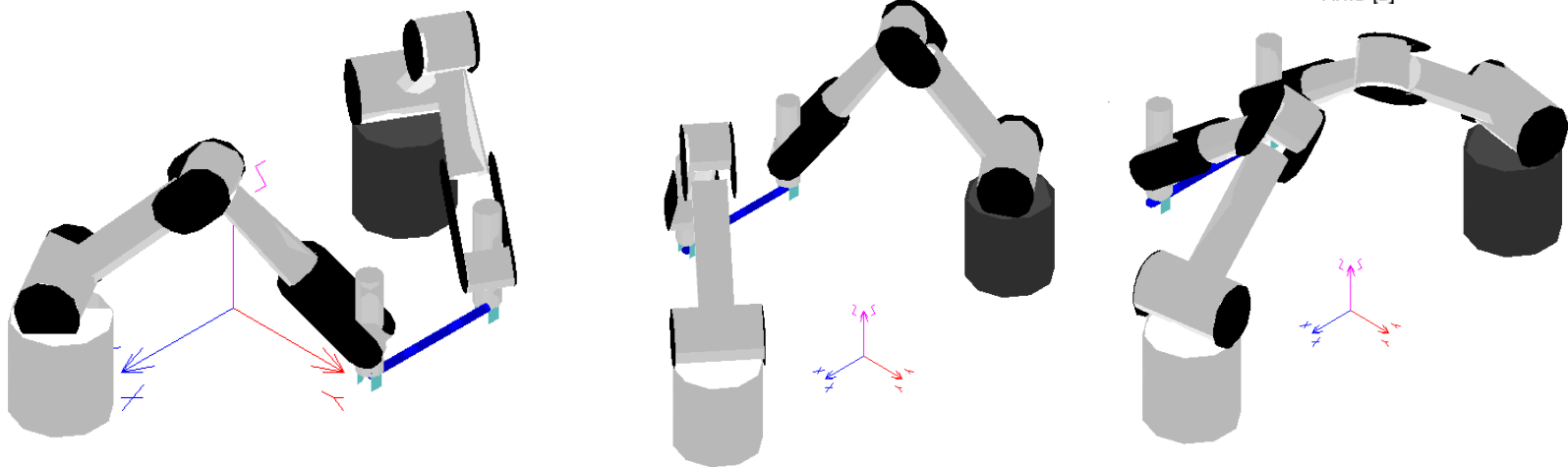
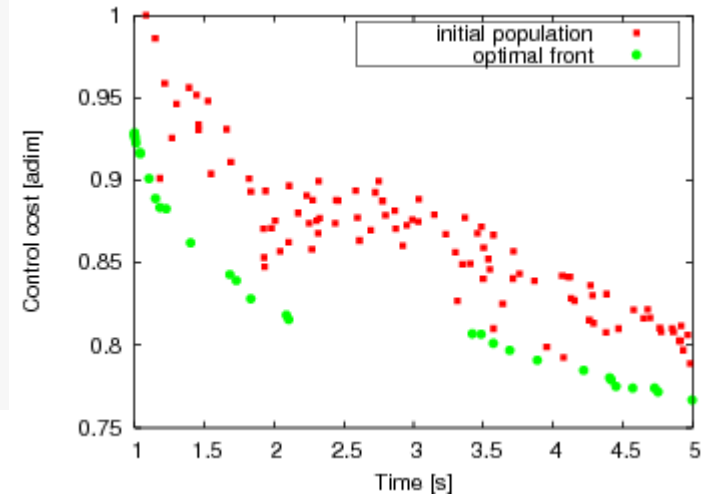
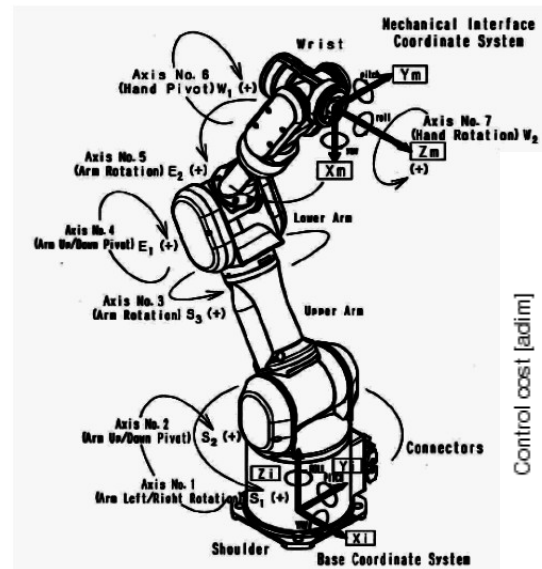


## inverse dynamics for computed torque control



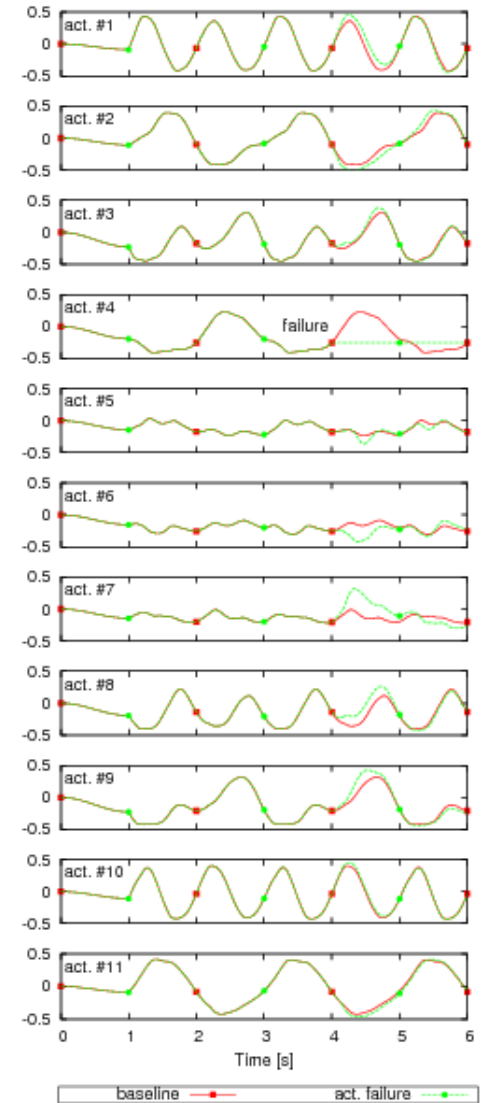
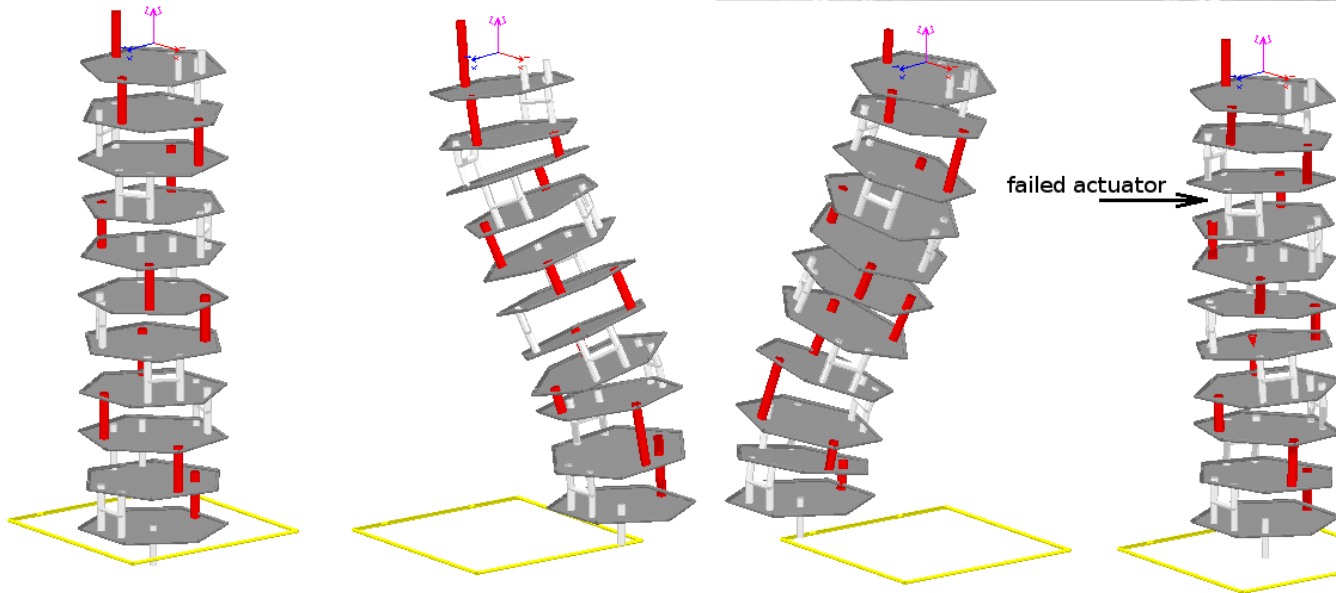
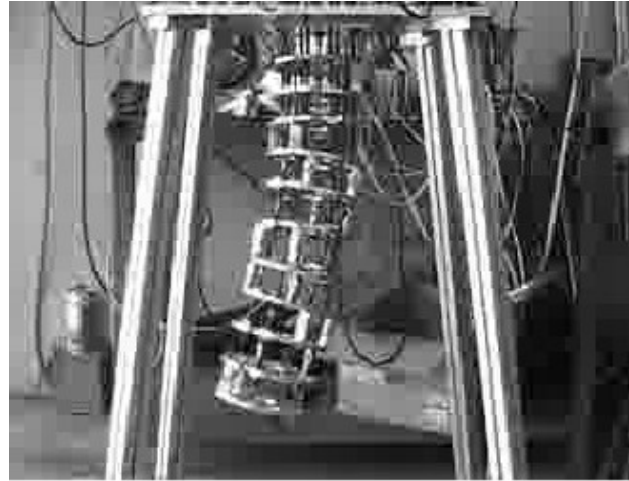
# Examples of multibody modeling with MBDyn

Robotics: PA-10  
inverse kinematics  
with path optimization  
of cooperating robots



# Examples of multibody modeling with MBDyn

**Robotics:**  
**biomimetic robot**  
**real-time motion**  
**planning by inverse**  
**kinematics with**  
**fault detection**





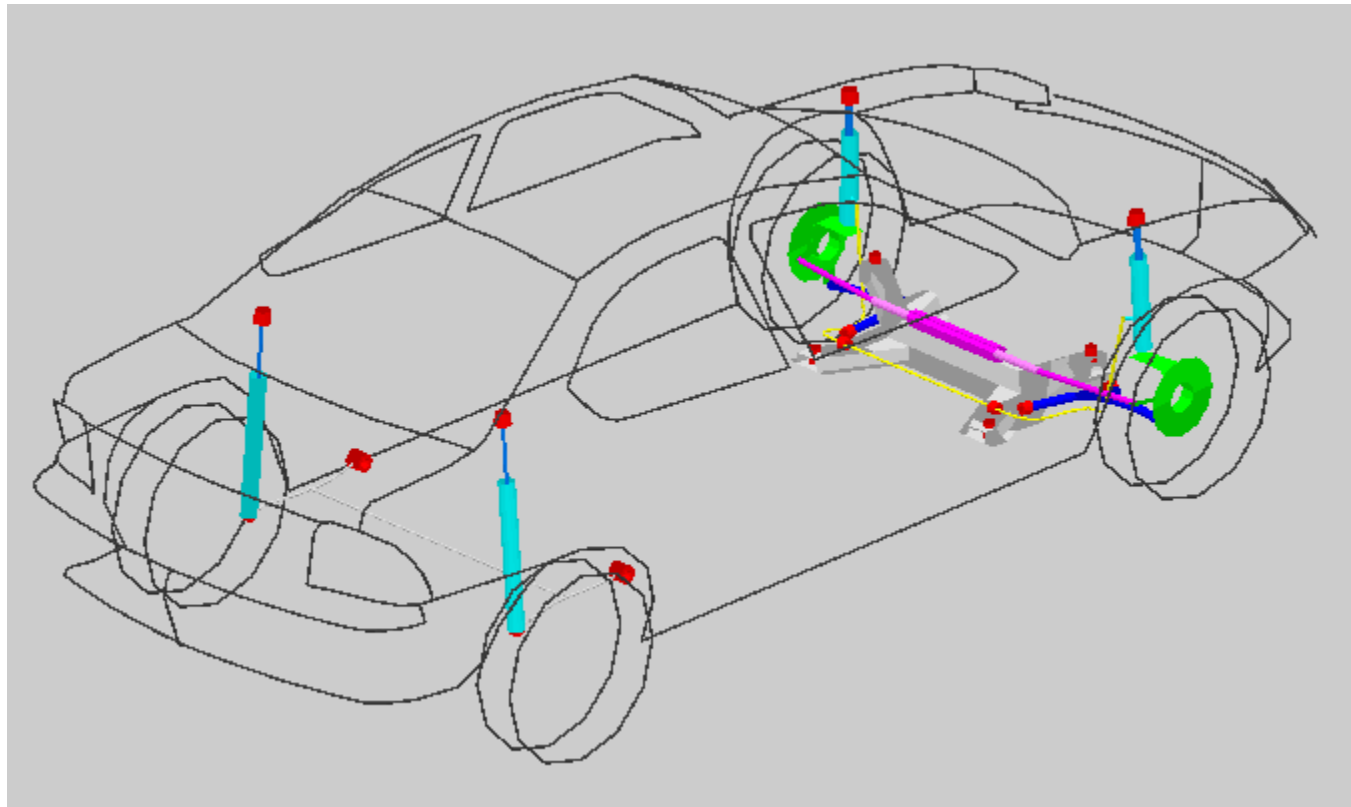
## **Industrial processes:**

- **simulation of automotive components assembly (car brake pipe) to:**
  - **check stresses introduced during assembly**
  - **check loads on supports introduced during assembly**
  - **check interference with other parts during assembly**
  - **check interference with other parts during operation**
- **the model has been developed by a rubber manufacturer**
- **it is used for product design and certification**
- **it required the development of specific features for solution partitioning, which are now part of MBDyn**

# Examples of multibody modeling with MBDyn



**Automotive: mechanical modeling of suspensions**  
**purpose: determine loads in rubber bushings and other components**

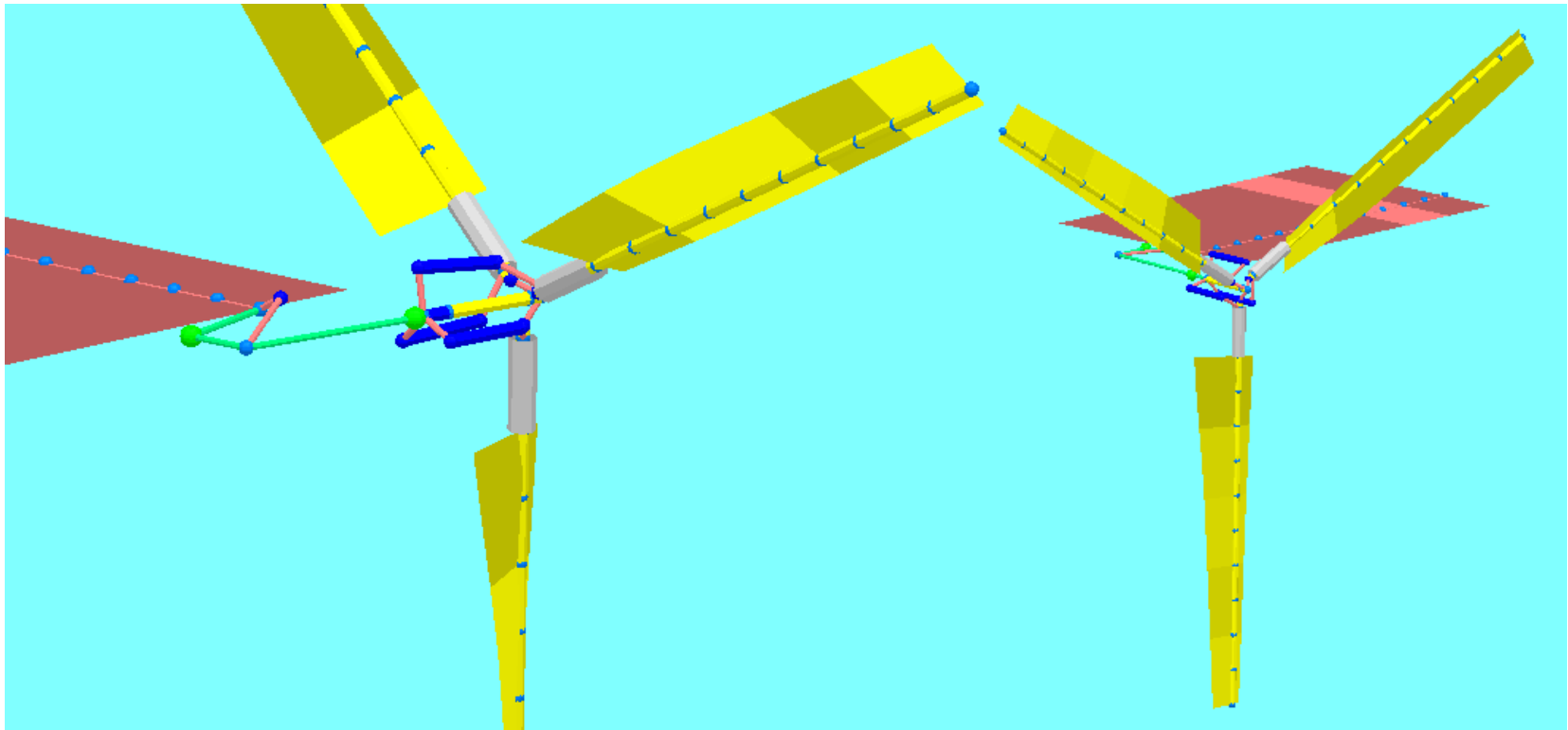


# Examples of multibody modeling with MBDyn



## Rotorcraft dynamics and aeroservoelasticity:

- WRATS (NASA/Army) tiltrotor aeromechanics

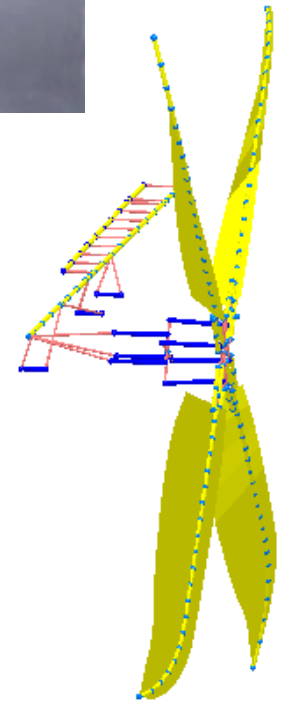
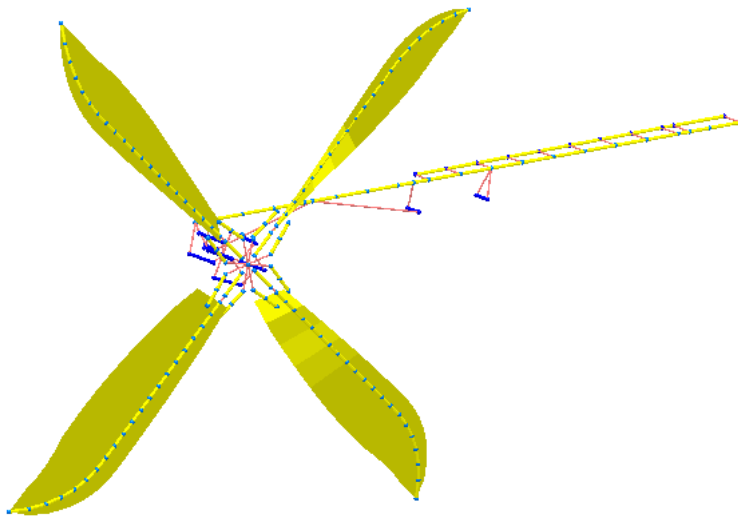


# Examples of multibody modeling with MBDyn



## Rotorcraft dynamics and aeroservoelasticity:

- ERICA (AgustaWestland) tiltrotor aeromechanics (ADYN, NICETRIP)





- **Multiscale handling of submodels with different dynamics**
  - aircraft flight mechanics (~1 to 5 Hz: very slow)
  - main rotor dynamics (~5 to 40 Hz: intermediate)
  - tail rotor dynamics (~25 to >100 Hz: fast)
- **Interfacing with different domains**
  - Fluid-structure (Lagrangian/Eulerian modeling of workflows)
  - structure-structure
  - active control of large deformable systems
- **Better abstraction/modularization of components/solution phases**
  - more freedom in model customization
  - tight integration into nonlinear structural analysis (Aster?)
- **More...**



- **Theory manual:**
  - **Incomplete; needs lots of work**
- **User manual**
  - **available and up to date**
- **Tutorials**
  - **available, but reportedly too simple; need work**
- **Applications manual**
  - **available, but only few applications so far**
- **Installation manual**
  - **available, incomplete and outdated (not critical)**
- **Mailing lists**
  - **available: announce, users, devel**
  - **the “users” list also serves as issue tracking provision**





- **Another important item that is missing is an automated test suite**
  - **can be run automatically after building the software**
  - **allows to check build errors**
  - **allows to check regressions in new releases**
  - **serves as example of modeling functionalities**
- **The rest is underway (always a work in progress)**

**Given the nature of the project, contributions are always welcome!**

A large graphic consisting of a light blue circle. Inside the circle, the text "RT-MBDyn" is written in a bold, dark blue, sans-serif font. Below the circle, the word "Questions?" is written in a bold, dark blue, sans-serif font.